

Neutron kinetics

This application note deals with a simple implementation of neutron kinetics for nuclear reactors fuelled with ^{235}U or ^{239}Pu respectively. It will be part of a little series of application notes culminating in the simulation of a nuclear power plant on an analog computer.

The neutron kinetics simulation is a central part of every nuclear reactor simulation.¹ Following [FENECH et al. 1973, p. 128] neutron kinetics can be described by

$$\dot{n} = \frac{n}{l^*}(\delta K - \beta) + \sum_i \lambda_i c_i + s \text{ and} \quad (1)$$
$$\dot{c}_i = \frac{n\beta_i}{l^*} - \lambda_i c_i.$$

Here, n is the neutron density, s represents an external neutron source (which is often used to “start” a nuclear reactor), l^* represents the effective neutron lifetime, which depends on the reactor design and geometry and can range between values as big as 10^{-2} s and 10^{-4} s.² c_i are the precursors of the i -th group of delayed neutrons, β_i represents the fraction of the i -th delayed neutron group, and λ_i is the decay constant for the i -th precursor group.

Typically, six groups of delayed neutron precursors are taken into account for a nuclear reactor simulation. Equation (1) could be programmed in a straightforward way but would require more effort using only standard computing elements. This application note shows how a specialized computing element can be devised that implements the neutron kinetics of a nuclear reactor with reasonable accuracy.

Figure 1 shows the basic structure of a such a computing element implementing the neutron kinetics for either a ^{235}U or ^{239}Pu reactor. The basic circuit consisting of

¹An overview on reactor simulation can be found in [MORRISON 1962]. Details of neutron kinetics and related topics are described in depth in [WEINBERG et al. 1958]. An actual implementation of a nuclear plant simulation is shown in [FENECH et al. 1973].

² s will be ignored in the following – an external neutron source can always be modelled by adding s to the input of the network described here.

RL and either CL1 or CL2 is that of an integrator with

$$l^* = RL \cdot CL_j, \quad 1 \leq j \leq 2.$$

Choosing $RL = 100 \text{ k}\Omega$ and $CL1 = 1 \text{ nF}$ yields $l^* = 10^{-4}\text{s}$. With the same RL a $CL2 = 100 \text{ nF}$ results in $l^* = 10^{-2}\text{s}$, two suitable and not too unrealistic values for the effective neutron lifetime in a nuclear reactor. The l^* to be used in a simulation run is selected by switch S1.

The six groups of delayed neutron precursors and resulting neutrons can be modelled by series circuits consisting of RU_i and CU_i for the case of ^{235}U and RP_i and CP_i for ^{239}Pu respectively. The capacitor values are determined according to³

$$C^*_i = \eta \frac{\beta_i}{\lambda_i}$$

with η being a scaling factor to get reasonable values for the capacitors. In this case $\eta = 10^2$ was chosen. With the capacitances determined this way, the resistors can be determined according to

$$R^*_i = \frac{1}{\eta \lambda_i C^*_i}.$$

Table 1 shows the λ_i and β_i for ^{235}U and ^{239}Pu with the corresponding values for the resistor-capacitor networks for both cases.⁴

The circuit shown in figure 1 can be used directly in an analog computer setup although it is advisable to add two electronic switches, one of which connects RL to the summing junction of the operational amplifier only when the computer is in operate mode, while the other switch discharges the selected capacitor $CL1$ or $CL2$ as well as the selected network for the delayed neutron groups when the computer is in initial condition mode.

³Cf. [FENECH et al. 1973, p.132]. C^*_i and R^*_i denote the capacitor and resistor values for either the ^{235}U or ^{239}Pu case.

⁴The values λ_i and β_i are according to [TYROR et al. 1970, p. 22]. Actual values tend to differ in the literature a bit.

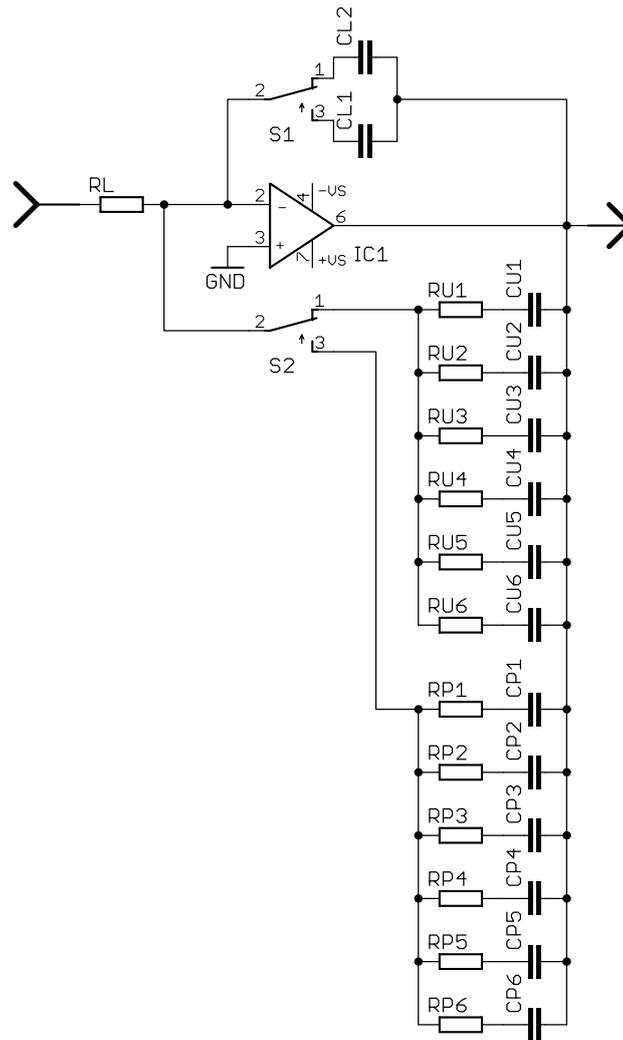


Figure 1: Basic circuit for the simulation of neutron kinetics with six groups of delayed neutrons

i	^{235}U				^{239}Pu			
	λ_i	β_i	$C_i = \frac{\eta\beta_i}{\lambda_i}$	$R_i [\Omega]$	λ_i	β_i	$C_i = \frac{\eta\beta_i}{\lambda_i}$	$R_i [\Omega]$
1	0.0127	0.000237	1.9 μF	414000	0.0129	0.0000798	619 nF	1252332
2	0.0317	0.001385	4.37 μF	72187	0.0311	0.000588	1.89 μF	170128
3	0.115	0.001222	1.06 μF	82034	0.134	0.0004536	339 nF	220138
4	0.311	0.002645	0.85 μF	37828	0.331	0.0006881	208 nF	145248
5	1.4	0.000832	59 nF	121065	1.26	0.0002163	17 nF	466853
6	3.87	0.000169	4.4 nF	587268	3.21	0.0000734	2.2 nF	1416029

Table 1: Delayed neutron data for ^{235}U and ^{239}Pu reactor

Due to the rather large values of the series connection of resistors and capacitors, the initial condition time should be chosen pretty large to make sure that the capacitors are suitably discharged.

References

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