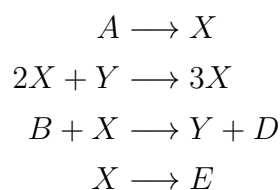




Analog Computer Applications

The Brusselator

The *Brusselator* is set of two coupled differential equations describing the following autocatalytic chemical reaction:¹



It is typically assumed that the reagents A and B are available in abundance, so their concentration does not noticeably change during the reaction. This simplified system can be described by the following two coupled differential equations:

$$\dot{x} = \alpha + x^2y - \beta x - x \quad (1)$$

$$\dot{y} = \beta x - x^2y \quad (2)$$

Typical values for α and β resulting in an oscillatory behavior of the system are $\alpha = 1$ and $\beta = 3$.

Before implementing this system on an analog computer equations (1) and (2) must be scaled. A good scaling factor is $\lambda = \frac{1}{5}$ for both x and y as a quick back-on-the-envelope calculation shows. Therefore scaled versions of α and β are introduced: $\alpha^* = \lambda\alpha = 0.2$ and $\beta^* = \frac{\beta}{10} = 0.3$. The latter is necessary as $\beta > 1$ in the standard parametrisation.

Taking this into account, equations (1) and (2) can now be rewritten as

$$\begin{aligned}x &= - \int 10\beta^*x - \alpha^* + x - \frac{x^2y}{\lambda^2} dt - 1 \text{ (initial condition) and} \\y &= - \int -10\beta^*x + \frac{x^2y}{\lambda^2} dt.\end{aligned}$$

¹The fascinating BELOUSOV-ZHABOTINSKY reaction is a practical example of this type of reaction.

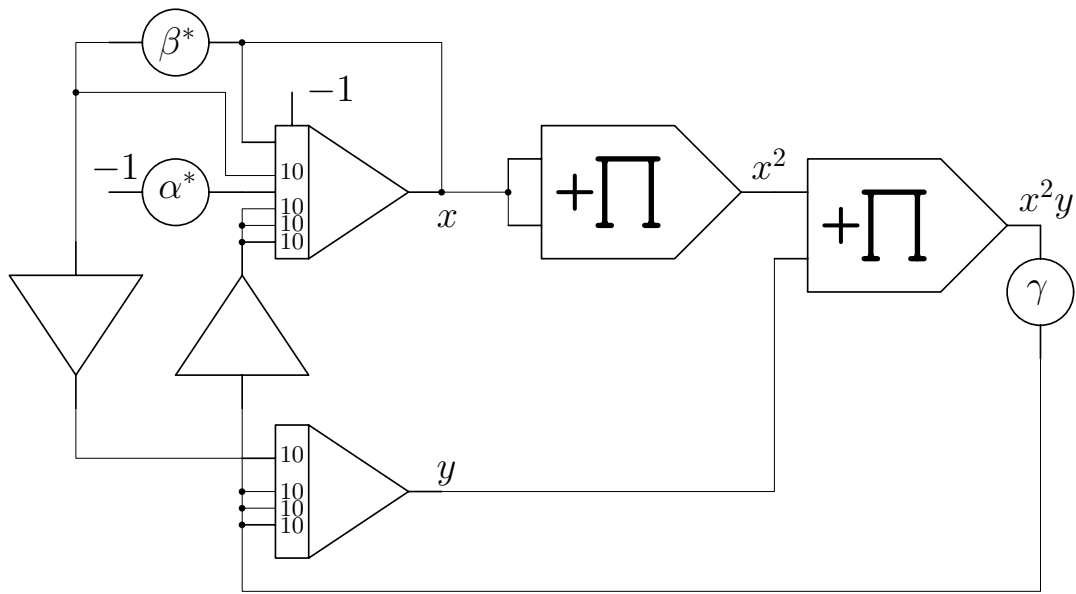


Figure 1: Analog computer program implementing the Brusselator

$\frac{1}{\lambda^2} = 25$ is actually implemented by paralleling three integrator inputs with weight 10 giving 30 and an additional scaling factor $\gamma = \frac{25}{30} \approx 0.833$.

Figure 1 shows the overall analog computer setup for the Brusselator. Since the integrators of THE ANALOG THING only have two inputs with weight 10, each integrator makes use of one of the two free resistor arrays (XIR) on THE ANALOG THING. To use these, the summing junction (SJ) of an integrator is connected with the SJ-jack of the XIR.

The actual implementation of this program on THE ANALOG THING is shown in figure 2 while a typical simulation result is shown in figure 3.²

²The oscilloscope used is a Hantek 6022 BE with OpenHantek (<http://openhantek.org>) running on a Mac.

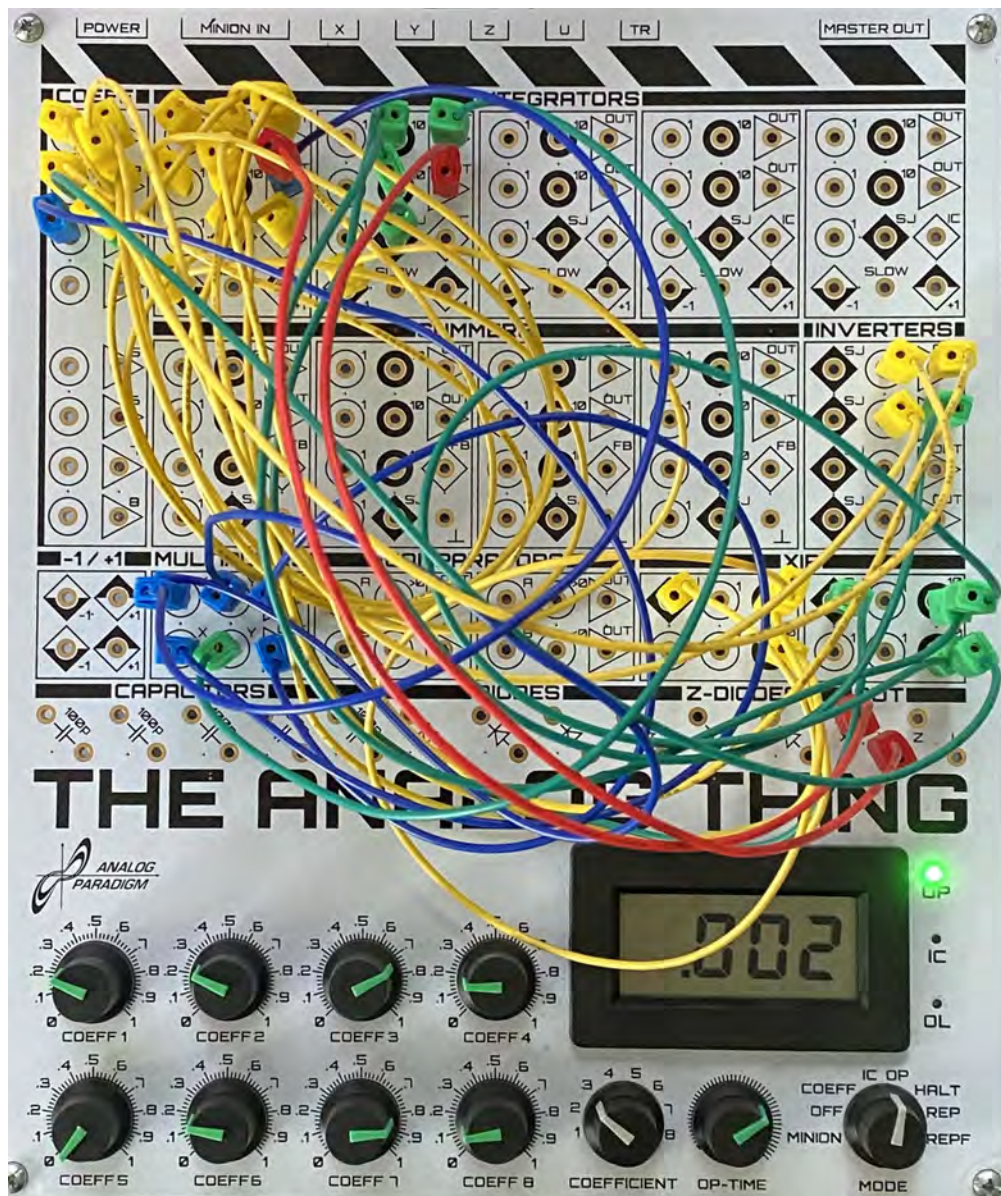


Figure 2: Implementation of the Brusselator program on THE ANALOG THING

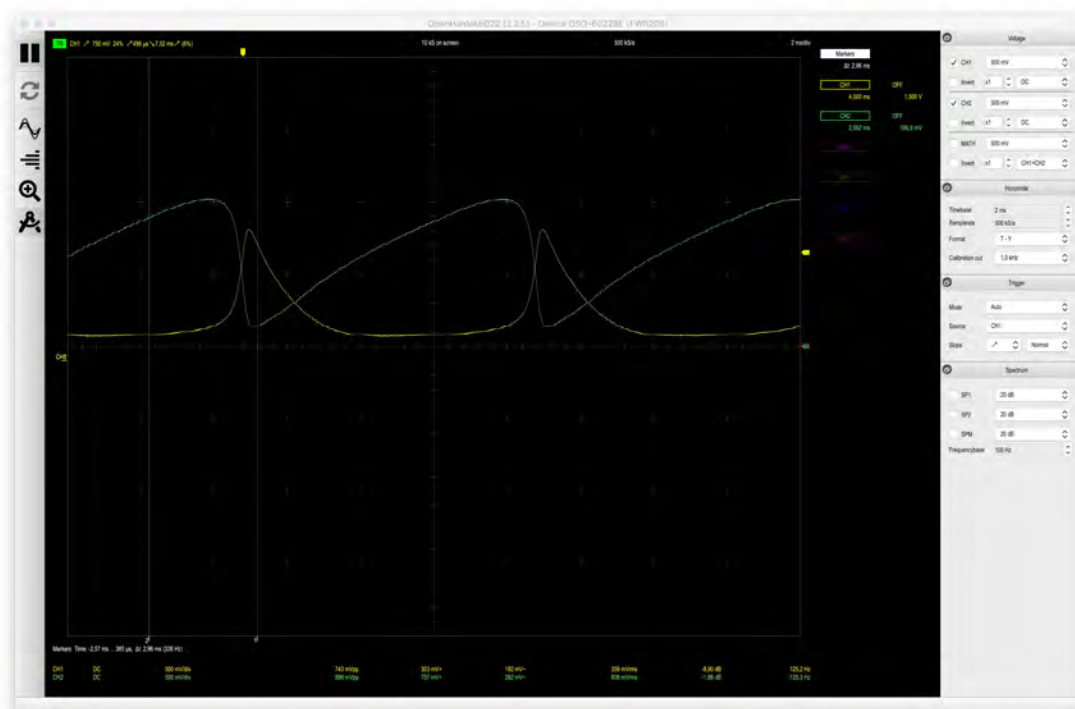


Figure 3: Typical simulation result