Ballistic trajectory

The following example computes the two-dimensional ballistic trajectory of a projectile fired from a cannon, taking the velocity dependent drag slowing the projectile down into account.[1] This drag causes the trajectory to deviate from a simple symmetric parabola as it will be steeper on its trailing half than on its leading half. The drag $\delta(v)$ is assumed to be of the general form

$$\delta(v) = rv^2 = r\left(\sqrt{\dot{x}^2+\dot{y}^2}\right)^2$$

which is a bit oversimplified but will suffice for the following. The general equations of motion of the projectile in this two-dimensional problem are

$$\ddot{x} = -\frac{\delta(v)}{m}\cos(\varphi) \text{ and} \tag{1}$$

$$\ddot{y} = -g - \frac{\delta(v)}{m}\sin(\varphi) \tag{2}$$

with g representing the acceleration of gravity, v denoting the projectile's velocity, and m being its mass. Obviously, it is

$$\cos(\varphi) = \frac{\dot{x}}{v} \text{ and}$$

$$\sin(\varphi) = \frac{\dot{y}}{v}.$$

Setting the mass $m := 1$ and rearranging (1) and (2), we get the

[1] Cf. [Korn, 1966, pp. 2-7 ff.].

following set of differential equations:

$$\ddot{x} = -\frac{\delta(v)}{v}\dot{x}$$
$$\ddot{y} = -g - \frac{\delta(v)}{v}\dot{y}$$

The computer setup resulting from these equations is shown in figures 1 and 2. The upper and lower halves of the circuit are symmetric except for the input for the gravitational acceleration to the lower left integrator yielding $\dot{y}$. The velocities $\dot{x}$ and $\dot{y}$ are fed to two multipliers yielding their respectives squares which are then summed and square rooted to get v as we have $\delta(v)/v = rv$.

The parameters α_1 and α_2 are scaling parameters that are set in order to get a suitably scaled picture on an oscilloscope operated in xy-mode. Table 1 shows the parameter set yielding the result shown in figure 3. The initial conditions satisfy

$$\dot{x}_0 = \cos(\varphi_0) \text{ and}$$
$$\dot{y}_0 = \sin(\varphi_0)$$

with φ denoting the elevation of the cannon. In this example $\varphi_0 = 60°$ has been chosen.

References

[Korn, 1966] Granino A. Korn, *Random-Process Simulation and Measurement*, McGraw-Hill Book Company, 1966

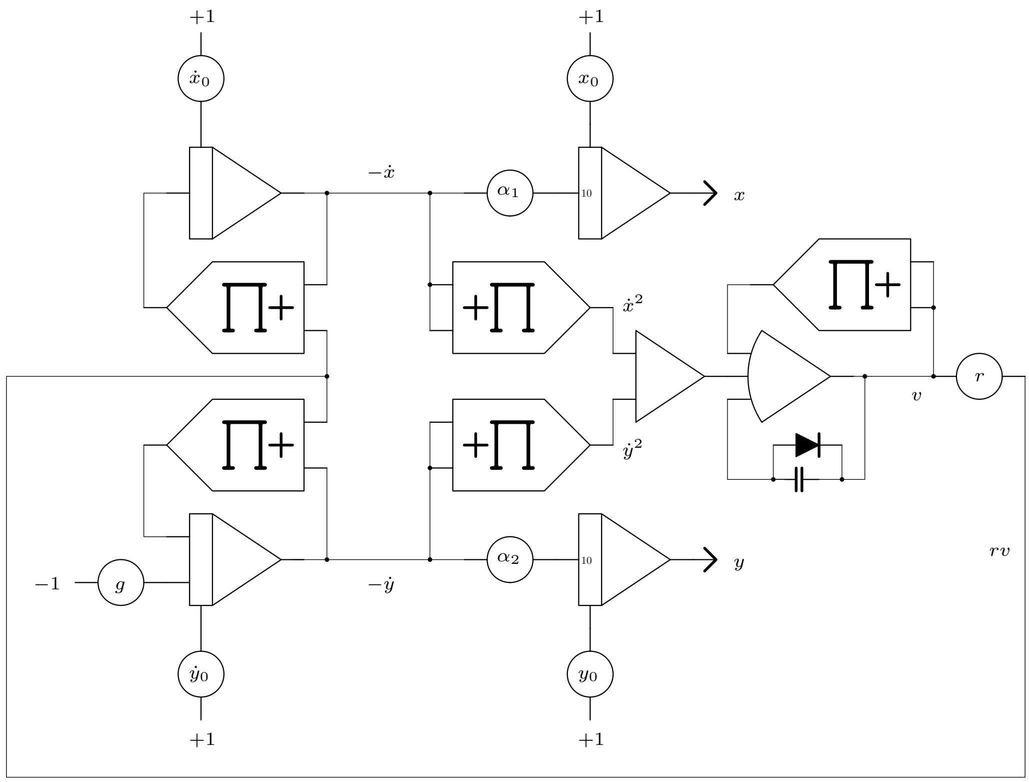

Figure 1: Computer setup for the simulation of a ballistic trajectory

Parameter	Value
$\dot{x}_0$	0.5
x_0	1
$\dot{y}_0$	0.86
y_0	1
g	0.72
r	1
α_1	0.34
α_2	0.55

Table 1: Parameter settings for the ballistic trajectory problem

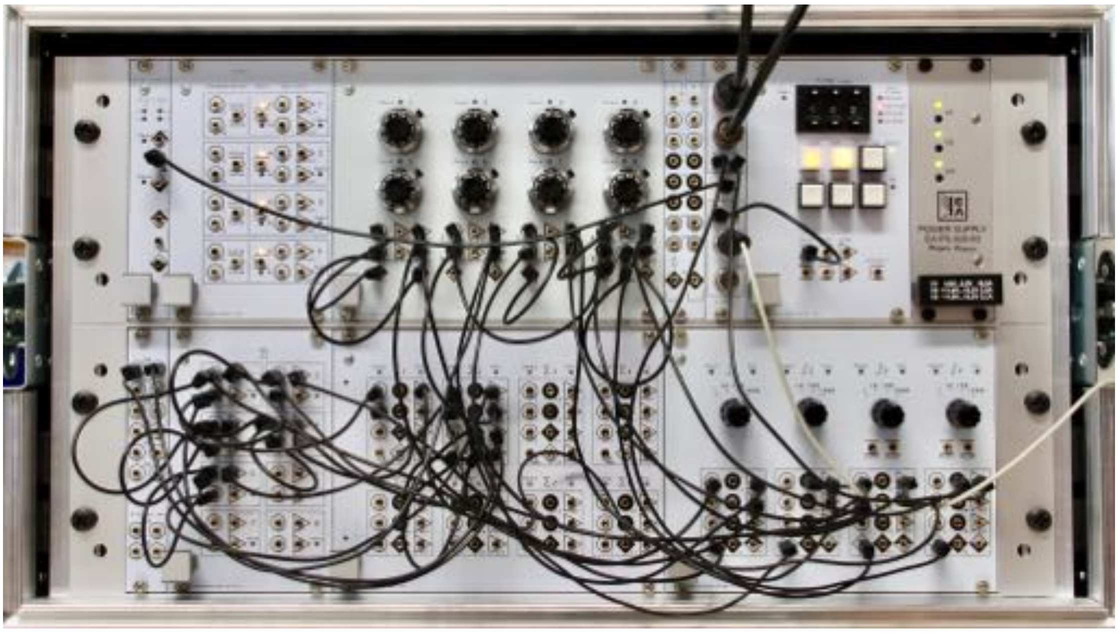

Figure 2: Setup of the ballistic trajectory problem on an Analog Paradigm Model-1 analog computer

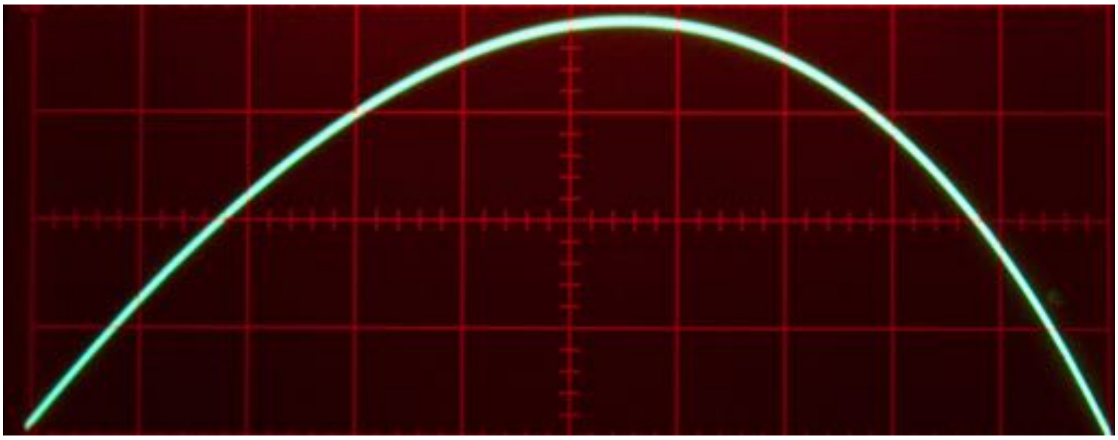

Figure 3: Ballistic trajectory