

Elegant Chaos

1 Introduction

One of the most intriguing books on chaotic systems is [SPROTT 2010] which is a true treasure trove for everybody fascinated by these mathematical objects. In the following, we will implement two of these systems on an analog computer.

2 A jerk system

So-called *jerk systems* are special cases of *autonomous dissipative systems*. The system under consideration in the following is described by the following differential equation:¹

$$\ddot{x} = -\ddot{x} + 9\dot{x} - x^2\dot{x} - 5x$$

Scaling chaotic systems for implementation on an analog computer is typically a bit nasty which holds true for this particular case, too. A few numerical calculations using a simple EULER integration routine show that the term $x^2\dot{x}$ has a domain exceeding $[-10^3 : 10^3]$ etc. Scaling is done by scaling down all inputs of one integrator by the same scale factor λ_i until no overload occurs anymore. Its output signal is then scaled back by $\frac{1}{\lambda_i}$ before being fed to the next integrator, which is then scaled as well etc. Table 1 shows the values for the six coefficient potentiometers required in the computer setup shown in figure 1. The

¹See [SPROTT 2010, p. 77].

Potentiometer	Calculated	Manually obtained
P1	0.1	0.1
P2	0.75	0.75
P3	0.12	0.17
P4	0.2	0.2
P5	0.333	0.1
P6	0.333	1

Table 1: Potentiometer settings for the jerk system

left column shows the parameters obtained by scaling “by the book” while the right column shows a parameter set that was obtained by tweaking some of the parameters manually. It is remarkable that the behaviour of the analog simulation shown in figure 2 is much more tame over a rather wide range of parameters than the numerical results shown in [SPROTT 2010, p. 77]. Nevertheless, it is possible to find regions that exhibit chaotic behavior by some manual parameter variation.

3 Nosé-Hoover Oscillator

Another beautiful chaotic system is the NOSÉ-HOOVER oscillator, an example of an autonomous conservative system² which is described by the following set of coupled differential equations:

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= yz - x \\ \dot{z} &= 1 - y^2\end{aligned}$$

²Cf. [SPROTT 2010, pp. 95 ff.].

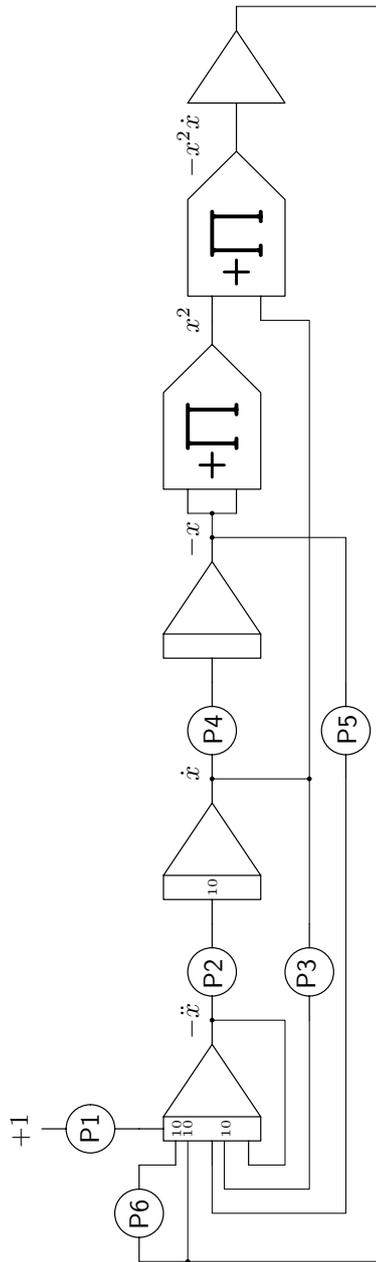


Figure 1: Setup for the jerk system

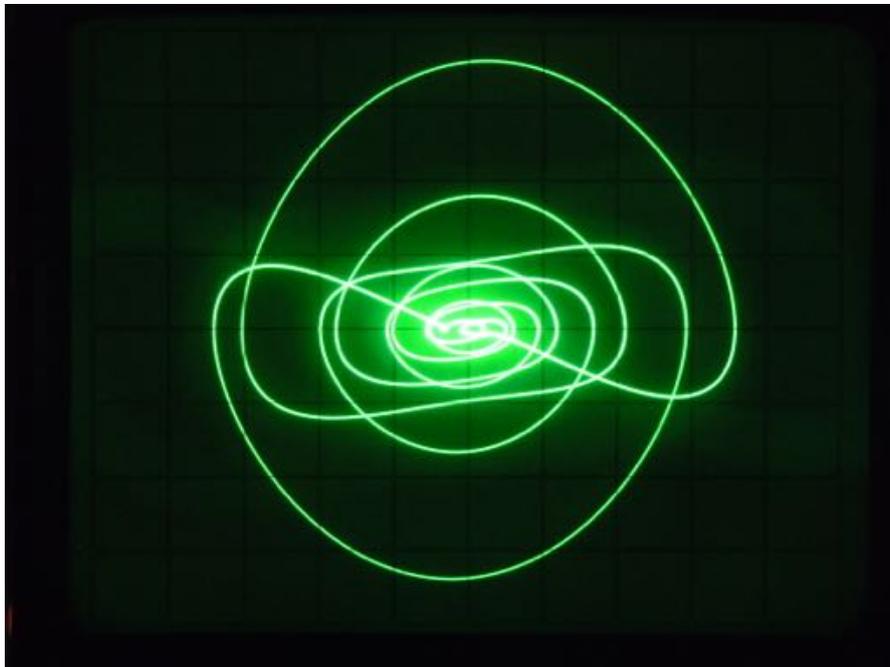


Figure 2: Typical behavior of the jerk system



Analog Computer Applications

Scaling this system is quite mean, too, and yields the computer setup shown in figure 3, the results of which are shown in the mesmerizing oscilloscope screen capture shown in figure 4.

References

[SPROTT 2010] JULIEN CLINTON SPROTT, *Elegant Chaos – Algebraically Simple Chaotic Flows*, World Scientific Publishing Co. Pte. Ltd, 2010

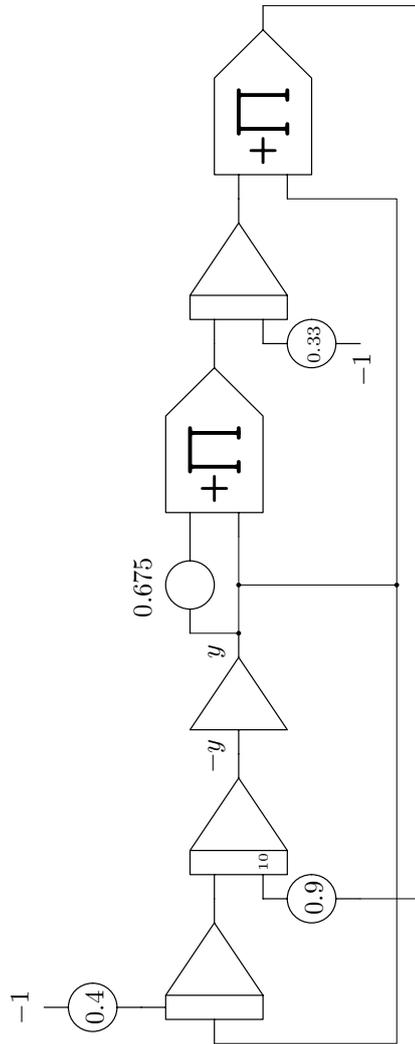


Figure 3: Setup for the NOSÉ-HOOVER oscillator

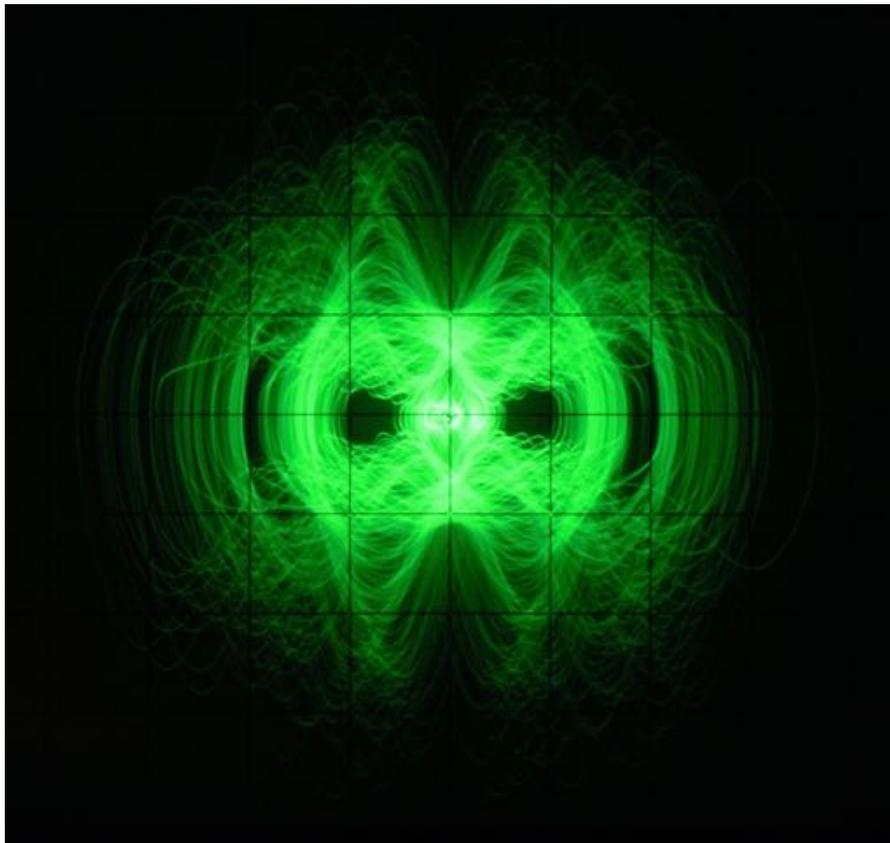


Figure 4: Phase state plot of the NOSÉ-HOOVER oscillator