

RUTHERFORD-scattering

1 Introduction

In the early 20th century, ERNEST RUTHERFORD performed a seminal experiment in which he bombarded a thin gold foil with α -particles. He observed that the vast majority of these particles, actually Helium nuclei, just went through the gold foil without being deflected or absorbed. Nevertheless, some of these particles were deflected through large angles, with even fewer α -particles being reflected by the gold foil, a phenomenon called RUTHERFORD-scattering. This led to the planetary RUTHERFORD model of the atom which became the basis of the BOHR model.

The following analog computer simulation which was motivated by [BORCHARDT] models the behaviour of such an α -particle following a trajectory near the core of a gold atom. We can assume that the gold atom's nucleus is at a fixed position, particularly the point of origin to simplify the distance calculation for the α -particle. With the α -particle's position being (x, y) its distance to the gold nucleus is

$$r^2 = x^2 + y^2.$$

The force exerted on the α -particle is described by COULOMB's law

$$F = k_e \frac{q_1 q_2}{r^2}, \quad (1)$$

where $k_e = 8.9875 \cdot 109 \text{Nm}^2\text{C}^{-2}$ is COULOMB's constant, and q_1, q_2 are the magnitudes of the charges involved. Since the following simulation



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will be a qualitative one, we gather all constants, including masses in a value γ .

The force F acts proportionally to $\cos(\varphi)$ and $\sin(\varphi)$ on x and y with

$$\cos(\varphi) = \frac{x}{r} \text{ and } \sin(\varphi) = \frac{y}{r}.$$

Together with (1) and γ this yields the following two coupled differential equations governing the path of the α -particle:

$$\begin{aligned}\ddot{x} &= \gamma \frac{x}{r^3} \\ \ddot{y} &= \gamma \frac{y}{r^3}\end{aligned}$$

2 Programming

The analog computer setup for this problem is pretty straight-forward as shown in figure 1. Please note the explicit (additional) feedback-path for the summer adding x^2 and y^2 resulting in a scaling factor of $\frac{1}{2}$. Thus the resulting sum always satisfies $0 \leq r_* \leq 1$, so no overload condition may occur here. As mentioned above, all constants (including the above factor $\frac{1}{2}$) are gathered in the parameter γ_* . A typical value for this parameter is ≈ 0.01 yielding a nice scattering behaviour.

The only other free parameter is the initial height of the α -particle's trajectory, y_0 . This can be either set manually by means of a free potentiometer, connected to +1 and -1, or automatically as shown in figure 2. Here, an integrator in conjunction with two comparators is setup as a triangle wave generator.

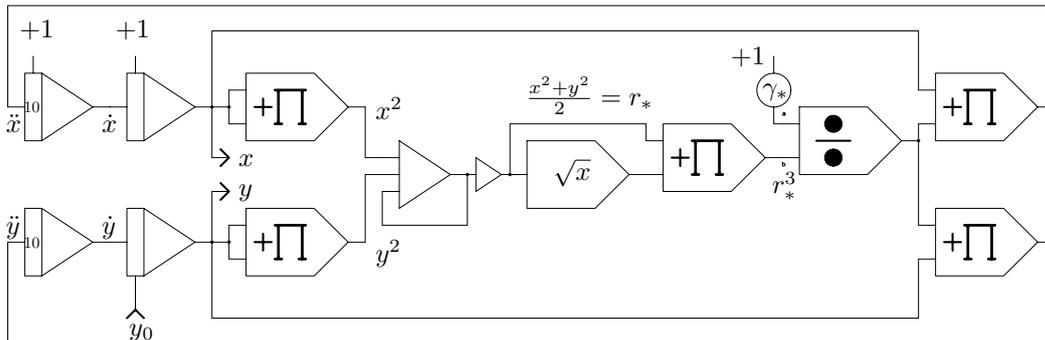


Figure 1: Simulation program for RUTHERFORD-scattering.

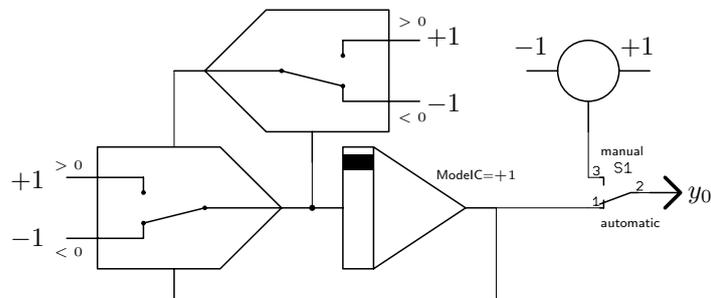


Figure 2: Automatic/manual generation of y_0



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3 Results

To run this program, the analog computer is set to repetitive operation with an operation time $t_{OP} = 30$ ms. The integrator of the triangle wave generator must be exempted from the central timing control by patching its ModelC-input to +1 (so only the third or fourth integrator of an INT4 module can be used for that purpose).

The picture shown in figure 3 was obtained with time-constants $k_0 = 100$ on the four integrators in figure 1, and $k_0 = 1$ in the triangle-wave generator. The camera was set to ISO 100 with an exposure time of 8 seconds.

References

[BORCHARDT] INGE BORCHARDT, *Kepler und die Atomphysik – Der Beschuss eines Atomkerns mit Alphateilchen auf einem Tischanalogrechner*, Demonstrationsbeispiel 3, AEG-Telefunken, ADB 003 0570

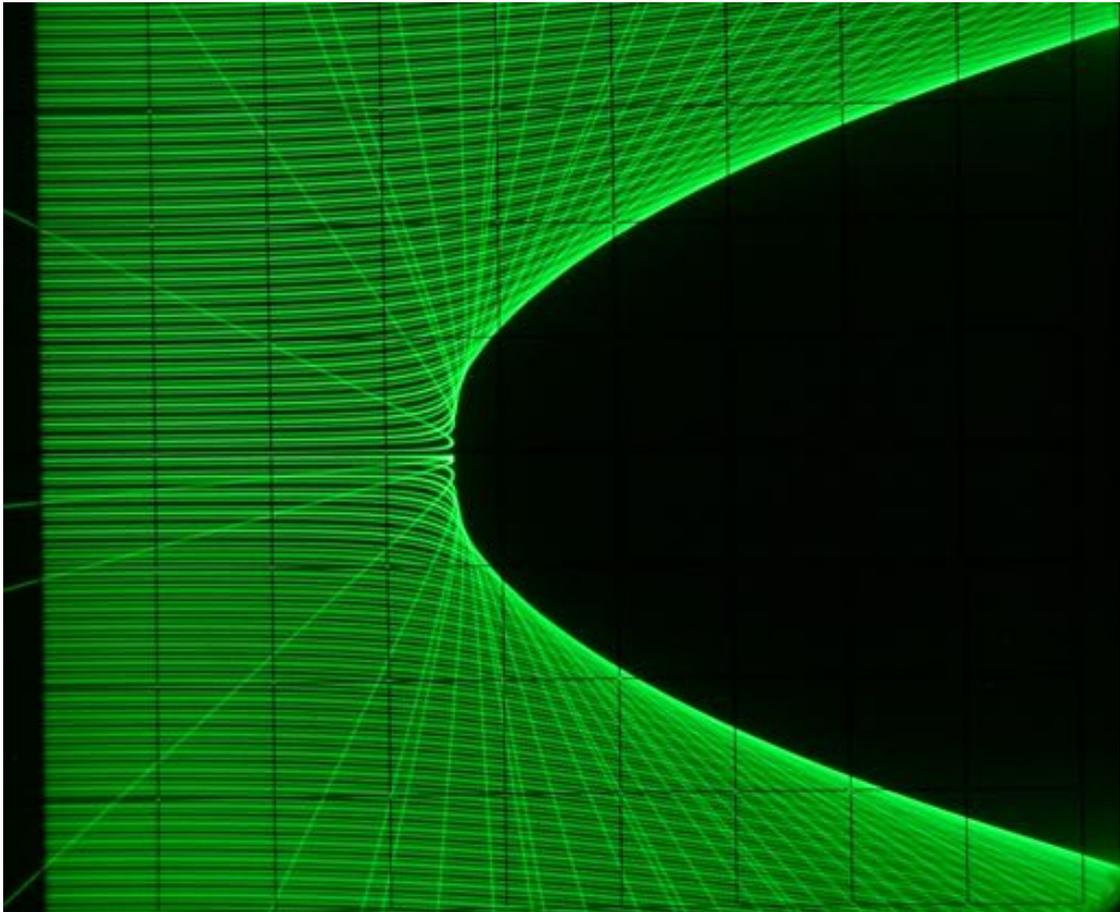


Figure 3: Simulation of RUTHERFORD-scattering