The Aizawa attractor

This application note implements one of the most beautiful chaotic attractors, the so-called Aizawa *attractor*.[1] The underlying system consists of three coupled differential equations,

$$\begin{aligned}\dot{x} &= x(z-\beta)-\delta y,\\ \dot{y} &= \delta x + y(z-\beta) \text{ and}\\ \dot{z} &= \gamma + \alpha z - \frac{z^3}{3} + \varepsilon z x^3\end{aligned}$$

with the parameters $\alpha = 0.095$, $\beta = 0.7$, $\gamma = 0.65$,[2] $\delta = 3.5$, and $\varepsilon = 0.1$. A thorough numerical study of the behavior of this particular system can be found in [Langford 1984].

Scaling these equations to ensure that no variable exceeds the machine units of ± 1 is pretty straight-forward: The scaling factors corresponding to x, y, and z are $\lambda_x = \frac{1}{3}$, $\lambda_y = \frac{1}{4}$, and $\lambda_z = \frac{1}{2}$. The value $z-\beta$ is scaled by $\frac{2}{5}$.

The resulting computer setup is a bit convoluted as shown in figure 1. All in all, three integrators, four summers, seven multipliers, and eleven coefficient potentiometers are required for this program.

Figure 2 shows the x, z phase space plot of the Aizawa attractor – an exceptionally beautiful structure. To achieve this, two of the parameters, resulting from the scaling, marked by $*$ and $**$ in figure 1, were varied a bit to achieve a "nicer" picture. The correct values for these are 0.95 and 0.27 respectively.

[1] See [Cope 2017].

[2] The original system has $\gamma = 0.6$ but 0.65 was determined as a better value.

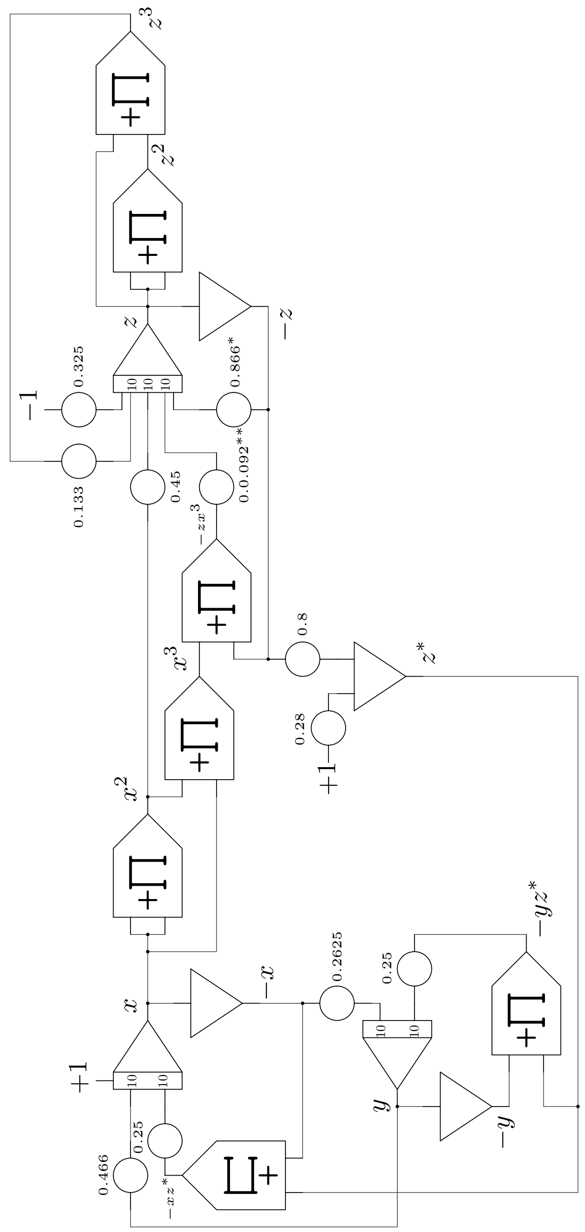

Figure 1: Program for the Aizawa attractor

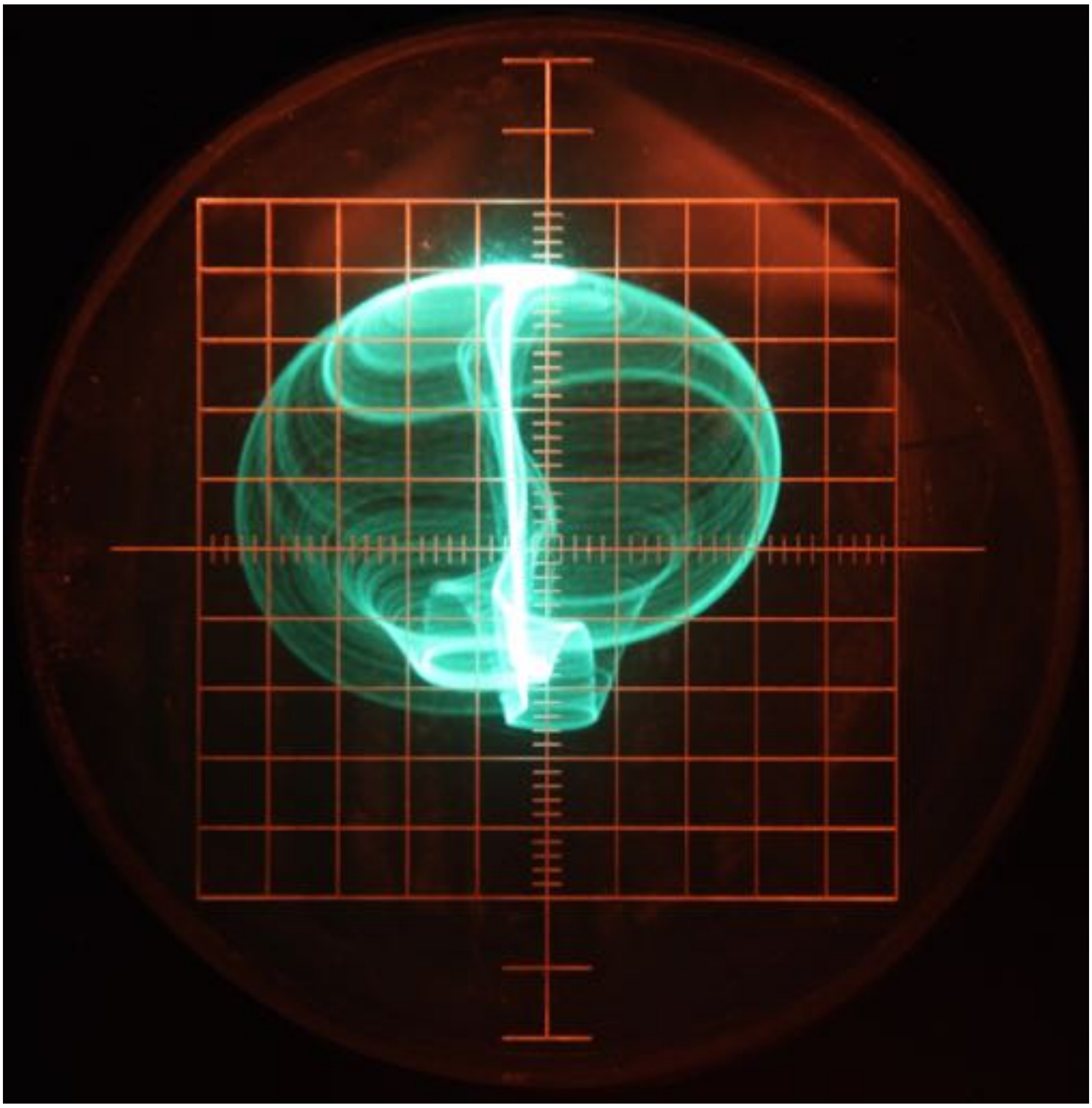

Figure 2: The x, z phase space plot of the AIZAWA attractor

References

[Cope 2017] Greg Cope, *The Aizawa Attractor*, http://www.algosome.com/articles/aizawa-attractor-chaos.html, retrieved 15.12.2018

[Langford 1984] William Finlay Langford, “Numerical Studies of Torus Bifurcations”, in International Series of Numerical Mathematics, Vol. 70, 1984 Birkhäuser Verlag Basel, pp. 285–295